Quarterly Journal of
Optoelectronic
Year 2, No. 3 (Serial Number 7) Winter 2018 (P 31-36)

فصلنامه
اپتوالکترونیک
سال دوم، شماره سوم (پیاپی 7)، زمستان 1396 (ص 31 - 36)

دینامیک انتشار پالس در فیبر بلور نوری با هسته تو خالی پر شده با گاز

پارسا زمانی *
استادیار، فیزیک، دانشگاه یاسوج

تاریخ دریافت: 1396/10/19 **تاریخ پذیرش**: 1398/07/02

Dynamics of Pulse Propagation in Gas-Filled Hollow-Core Photonic Crystal Fibers

P. Zamani*
Assistant Professor, Physics, Yasuj University

Received: 2018/01/09 **Accepted:** 2019/08/24

Abstract
As a new approach based on a multiple scale perturbation theory, we have founded clearly that ionization in a hollow-core photonic crystal fiber filled by an ionizable gas leads to a soliton self-frequency blueshift, opposite to redshift due to Raman self-frequency. To analyze the hollow-core photonic crystal fiber under a general perturbation, we convert the problem into a system of ordinary differential equations which govern the soliton parameters. It is worth comparing the results with those of Saleh et al [Phys.Rev.Lett. 107, 203902 (2011)] applying a variational perturbation method with a posteriori. We have obtained just the same result with no posteriori.

Keywords
Hollow-Core Photonic Crystal Fiber, Multiple Scale Perturbation Theory, Pulse Propagation, Soliton, System of Ordinary Differential Equations.

چکیده
به عنوان روشی جدید بر اساس نظریه اختلال چند مقیاسی نشان می‌دهیم که یونیزاسیون در فیبر بلور نوری با هسته تو خالی پر شده با گاز قابل یونیزه منجر به شیفت آبی خود-فرکانسی سالیتونی می‌شود که در تضاد با شیفت قرمز وابسته به خود-فرکانسی رامانی است. برای تحلیل فیبر بلور نوری با هسته تو خالی تحت یک اختلال کلی، مسئله را به سیستمی از معادلات دیفرانسیل معمولی حاکم بر پارامترهای سالیتون تبدیل می‌کنیم. نتایج به دست آمده دقیقا همان نتایجی هستند که توسط صالح و همکاران در سال 2011 با استفاده از روش اختلال وردشی با اعمال یک پیش فرض به دست آمده است. ما این نتایج را بدون اعمال هیچ گونه پیش فرضی به دست آورده‌ایم.

واژگان کلیدی
فیبر بلور نوری با هسته تو خالی-نظریه اختلال چند مقیاسی - انتشار پالس – سالیتون - سیستم معادلات دیفرانسیل معمولی.

*Corresponding Author: p_zamani@yu.ac.ir

* **نویسندهٔ مسئول**: پارسا زمانی

Introduction

Hollow-core photonic crystal fibers (HC-PCFs) have been the focus of a number of studies over the last years [1,2,3,15,16]. HC-PCFs represent a kind of silica waveguides with the central air core surrounded by the microstructured cladding of periodic air hole. An important advantage of HC-PCFs is that light is guided in the hollow core, preparing a strong background for investigating light-matter interactions when these fibers are filled with a gas [4,5]. HC-PCFs with the so-called kagome-lattice have experienced considerable breakthrough in nonlinear optics. Via filling air holes with different gases, new phenomena and applications in these fibers can be demonstrated. Therefore the nonlinearity and the group velocity dispersion can be varied by changing the gas pressure, allowing the control of many important nonlinear effects in the gas such as stimulated Raman scattering, soliton dynamics and so on [6,7]. Recently, these fibers have successfully been used to demonstrate a limited ionization-induced blue-shift of guided ultrashort pulses [8].

Saleh et al applied a model to study the pulse propagation in gas-filled hollow-core photonic crystal fibers which neglect losses for the first time [9]. To obtain the results, they were required to apply a posteriori. They assumed that the soliton functional shape to be unchanged during the action of the perturbation.

We start with the model presented by Saleh et al, apply a new method, multiscale perturbation method and derive the system of coupled equations govern evaluation of the soliton parameters. It is possible to obtain just the same results and, thus, there is no need for posteriori to obtain the results. To map out the contents of the present study, in section 2 we introduce the model proposed by Saleh et al. In section 3, the proposed approach is applied to the perturbed nonlinear Schrödinger equation and the coupled equations which govern the evaluation of soliton parameters are derived. This system of ordinary differential equations can be solved by the analytical or numerical methods. In section 4 we compare our solutions with those obtained by Saleh et al in [9,10]. The paper is concluded by a conclusion section.

Physical Background

In an HC-PCF filled with an ionized Raman-active gas, light propagation is described by the following coupled equations [9,10]:

$$\left[i\partial_z + \widehat{D}(i\partial_t) + \gamma_K R(t)\otimes|\Psi(t)|^2 - \frac{\omega_p^2}{2k_0c^2} + i\alpha\right]\Psi = 0 \qquad (1)$$

$$i\partial_t n_e = \left[\frac{\tilde{\sigma}}{A_{eff}}\right][n_T - n_e]\Delta|\Psi|^2\Theta(\Delta|\Psi|^2) \qquad (2)$$

Where $\Psi(z,t)$ is the electric field envelope, z is the longitudinal coordinate along the fiber, t is the time in reference frame moving with the pulse group velocity, $\widehat{D}(i\partial_t) \equiv \sum_{m\geq 2}\beta_m (i\partial_t)^m/m!$ is the full dispersion operator, β_m is the mth order dispersion coefficient calculated at an arbitrary reference frequency ω_0, γ_K is the Kerr nonlinear coefficient of gas, $R(t) = (1-\rho)\delta(t) + \rho h(t)$ is the normalized Kerr and Raman response function of gas, $\delta(t)$ is the Dirac delta function, ρ is the relative strength of the non-instantaneous Raman nonlinearity, and $h(t)$ is the causal Raman response function of the gas [8,11]. The symbol $\otimes$ denotes the time convolution $[A\otimes B = \int A(t-t')B(t')dt' = \int B(t-t')A(t')dt']$, c is the speed of light, $k_0 = \omega_0/c$, ω_0 is the pulse central frequency, $\omega_p = [e^2 n_e/(\epsilon_0 m_e)]^{\frac{1}{2}}$ is the plasma frequency associated with an electron density n_e, e and m are the electron charge and mass, respectively. ϵ_0 is the vacuum permittivity; $\alpha = \alpha_1 + \alpha_2$ is the total loss coefficient, α_1 is the fiber loss, $\alpha_2 = \frac{A_{eff}U_I}{2|\Psi|^2}\partial_t n_e$ is the ionization-induced loss term, A_{eff} is the effective optical mode area; U_I is the ionization energy of the gas; $\Delta|\Psi|^2 = |\Psi|^2 - |\Psi|^2_{th}$, $|\Psi|^2 = IA_{eff}$, $|\Psi|^2_{th} = I_{th}A_{eff}$, and n_T is the total number density of ionizable atoms in the fiber, associated with the maximum plasma frequency $\omega_T = [e^2 n_T/(\epsilon_0 m_e)]^{\frac{1}{2}}$.

For pulses with maximum intensities just above the ionization threshold, the ionization loss is not large and can be neglected as a first approximation. Hence the two coupled equations can be replaced by [9, 10]:

$$\left[i\partial_\xi + \widehat{D}(i\partial_\tau) + R(\tau)\otimes|\psi(\tau)|^2 - \phi\right]\psi = 0 \quad (3)$$

$$\partial_t\phi = \sigma(\phi_T - \phi)|\psi|^2 \quad (4)$$

Where $\psi = \sqrt{\gamma_K z_0}\Psi$, $\xi = \frac{z}{z_0}$ and $\tau = \frac{t}{t_0}$ are normalized versions of the propagation distance z and t in a reference frame that move at group velocity, $r(\tau) = R(t)t_0$, $\phi = \frac{1}{2}k_0 z_0(\frac{\omega_p}{\omega_0})^2$; $\phi_T = \frac{1}{2}k_0 z_0(\frac{\omega_T}{\omega_0})^2$ represents the maximum plasma frequency ω_p, k_0 is the corresponding vacuum wavenumber and $\sigma = \frac{\tilde{\sigma}' t_0}{A_{eff}\gamma_K z_0}$ refers to photonization cross-section σ'. On these latter relations, $z_0 = \frac{t_0^2}{|\beta_2(\omega_0)|}$ is the second order dispersion length at the reference frequency ω_0 , β_2 is the second order dispersion coefficient and t_0 is the input pulse duration.

The second equation [4] can be solved analytically, $\phi(\tau) = \phi_T\{1 - e^{-\sigma\int_{-\infty}^{\tau}|\psi(\tau')|^2 d\tau'}\}$ with the initial condition $\phi(-\infty) = 0$, corresponding to the absence of any plasma before the pulse arrives.

For a small ionization cross section the two coupled equations can be reduced to a single generalized nonlinear Schrödinger equation [9, 10]:

$$i\frac{\partial\psi}{\partial\xi} + \frac{1}{2}\frac{\partial^2\psi}{\partial\tau^2} + |\psi|^2\psi - \tau_R\psi\frac{\partial|\psi|^2}{\partial\tau} - \eta\psi\int_{-\infty}^{\tau}|\psi|^2 d\tau' = 0 \quad (5)$$

Where $\eta = \sigma\phi_T$ and $\tau_R \equiv \int_{\mathbf{0}}^{\infty}\tau' r(\tau')d\tau'$. This equation is a perturbed nonlinear Schrödinger equation (**NLSE**) in which the fourth and fifth terms are the perturbation function. The fifth term includes a derivation of the field intensity and represents the Raman Effect while the fifth term includes an integral on the same equality and represents the ionization effect. In addition, this equation shows clearly that the effect of ionization is essentially opposite to that of the Raman Effect. In order to investigate the Raman and ionization effects on the dynamic of solitons, we first apply the multiscale perturbation theory for the perturbed **NLSE** in the general case.

Perturbation Theory for a Nonlinear Schrodinger Equation

A powerful approach to solving a large class of nonlinear systems with a very nice approximate solution is based on the multiscale perturbation theory [12]. We consider the perturbed **NLSE** as

$$i\frac{\partial\psi}{\partial\xi} + \frac{1}{2}\frac{\partial^2\psi}{\partial\tau^2} + |\psi|^2\psi = \epsilon R(\psi, \psi_\xi, \psi_\tau) \quad (6)$$

Where R is a small perturbation

In the absence of perturbation $\epsilon = 0$, the general solution of the **NLSE** contains soliton waves. A soliton is described by four parameters:

$$\psi(\xi,\tau) = A\,Sech[A(\tau - \delta\xi + \tau_0)]e^{i\left[\sigma\tau - \frac{1}{2}(\delta^2 - A^2)\xi + \phi_0\right]} \quad (7)$$

Where A, δ, τ_0 and ϕ_0 are constants and include physical information. These parameters represent amplitude, frequency, position and phase of the soliton, respectively [11]. In the presence of a small perturbation, the four soliton parameters will be varied. We seek a perturbation expansion ψ as follows,

$$\psi \cong \psi_0 + \epsilon\psi_1 + \cdots \quad (8)$$

Where ψ_0 denotes the N-soliton solution which satisfies the **NLSE**.

Since the weak perturbations affect a soliton, the soliton parameters will be varied. Therefore we introduce the temporal time and phase of soliton as:

$$\tau_p(\xi) = \int_0^{\xi}\delta(\xi')d\xi' - \tau_{0p}(\xi) \quad (9)$$

$$\phi(\xi) = -\frac{1}{2}\int_0^{\xi}[\delta^2(\xi') - A^2(\xi')]d\xi' + \phi_0(\xi) \quad (10)$$

Note. The important property,

$$\lim_{\epsilon\to 0} \epsilon\psi_1\left(\frac{\zeta}{\epsilon},\tau\right) = 0 \quad (11)$$

for fixed ζ, is called the secularity condition [12]. This condition claims that $\epsilon\psi_1$ remains small for space as large as $O(\epsilon^{-1})$. Due to applying the secularity condition, ψ_0 no longer satisfies **NLSE**. Instead,

$$i\frac{\partial\psi_0}{\partial\xi} + \frac{1}{2}\frac{\partial^2\psi_0}{\partial\tau^2} + |\psi_0|^2\psi_0 = O(\epsilon) \quad (12)$$

Substituting (8) into (6), we find that the results for the first-order ψ_1:

$$\left(i\frac{\partial}{\partial\xi} + \sigma_3 H\right)\begin{pmatrix}\psi_1\\ -\psi_1^\star\end{pmatrix} = \begin{pmatrix}R\\ -R^\star\end{pmatrix} - i\begin{pmatrix}\psi_{0\tau}\\ \psi_{0\tau}\end{pmatrix} \quad (13)$$

Where σ_3 is the third Pauoli spin matrix and the Hermitian matrix H is given by [13]:

$$H = \begin{pmatrix}\frac{1}{2}\frac{\partial^2}{\partial\tau^2} + 2|\psi_0|^2 & \psi_0^2\\ \psi_0^{2\star} & \frac{1}{2}\frac{\partial^2}{\partial\tau^2} + 2|\psi_0|^2\end{pmatrix} \quad (14)$$

For convenience we define,

$$\Phi = (\psi_1, \psi_1^\star)^T, P = (R, -R^\star)^T,$$
$$Q = i(\psi_{0\tau}, {\psi_{0\tau}}^\star)^T, \Psi = (\psi_0, {\psi_0}^\star)^T$$

Therefore Equation (13) can be written as

$$L\Phi = \mathrm{P} - \mathrm{Q} \quad (15)$$

It is clear that,

$$Q = i\sum_{j=1}^{4N}(A_{j\xi}\,\psi_{A_j} + \tau_{0pj\xi}\psi_{\tau_{0pj}} + \delta_{j\xi}\psi_{\delta_j} + \phi_{0j\xi}\psi_{\phi_{0j}}) \quad (16)$$

We only need to study the null space of the operator $L = i\frac{\partial}{\partial\xi} + \sigma_3 H$. This space consists of two explicit discrete and continuous subspaces. The continuous subspace with eigenfunctions $\Phi_c(\xi,\tau,\lambda)$ is related to dispersive waves while the discrete subspace is related to solitons. The discrete and continuous eigenfunctions form a complete set for linear equation(15) [13]. The discrete component of L is 4N-dimensional and is spanned by the set,

$$\left\{\frac{\partial\Psi}{\partial A_j}, \frac{\partial\Psi}{\partial\delta_j}, \frac{\partial\Psi}{\partial\tau_{0j}}, \frac{\partial\Psi}{\partial\phi_{0j}}, j = 1,2,\dots N\right\} \quad (17)$$

Due to this completeness, the inner product is defined by:

$$\langle\psi_i, \psi_j\rangle = \int_{-\infty}^{+\infty}\psi_i^{*T}\sigma_3\,\psi_j d\tau \quad (18)$$

We can expand the solution Φ and forcing function $P - Q$ into this complete set:

$$\Phi = \sum_{j=1}^{4N}(a_{1j}\,\psi_{A_j} + a_{2j}\psi_{\tau_{0pj}} + a_{3j}\psi_{\delta_j} + a_{4j}\psi_{\phi_{0j}}) + \int C_\lambda\Phi_c\,(\xi,\tau,\lambda)d\lambda \quad (19)$$

$$\mathrm{P} - \mathrm{Q} = \sum_{j=1}^{4N}(b_{1j}\,\psi_{A_j} + b_{2j}\psi_{\tau_{0pj}} + b_{3j}\psi_{\delta_j} + b_{4j}\psi_{\phi_{0j}}) + \int D_\lambda\Phi_c\,(\xi,\tau,\lambda)d\lambda \quad (20)$$

To obtain the equations which determine the evaluation of soliton parameters, according to the procedure that has been used by Yang [14], we find the nonzero inner products of the discrete eigenfunctions as:

$\langle\psi_{\tau_{0j}}, \psi_{\delta_j}\rangle = -2iA_j$
and
$\langle\psi_{A_j}, \psi_{\phi_{0j}}\rangle = 2i$

The next step is to form the inner products of set $\{\psi_{A_j}, \psi_{\delta_j}, \psi_{\tau_{0j}}, \psi_{\phi_{0j}}, j = 1,2,\dots,N\}$ and Eq.(20). In order to avoid the secular terms which will invalidate the perturbation expansion, we enforce the condition $b_{kj} = 0$.

Therefore, we can derive the system of coupled equations which govern the evaluation of the soliton parameters as follows,

$$A_{j\xi} = -\frac{\epsilon}{2}\langle P, \psi_{\phi_{0j}}\rangle \qquad (21)$$

$$\tau_{0j\xi} = \frac{\epsilon}{2A_j}\langle P, \psi_{\delta_j}\rangle \qquad (22)$$

$$\delta_{j\xi} = -\frac{\epsilon}{2A_j}\langle P, \psi_{\tau_{0j}}\rangle \qquad (23)$$

$$\phi_{0j\xi} = -\frac{\epsilon}{2}\langle P, \psi_{A_j}\rangle \qquad (24)$$

For a single soliton we find that,

$$A_\xi = -\frac{\epsilon}{2}\langle P, \psi_{\phi_0}\rangle \qquad (25)$$

$$\tau_{p\xi} = -\frac{\epsilon}{2A}\langle P, \psi_\delta\rangle + \delta \qquad (26)$$

$$\delta_\xi = -\frac{\epsilon}{2A}\langle P, \psi_{\tau_0}\rangle \qquad (27)$$

$$\phi_\xi = -\frac{1}{2}(\delta^2 - A^2) + \frac{\epsilon}{2}\langle P, \psi_A\rangle \qquad (28)$$

Here,

$$\langle P, \psi\rangle = 2Re\int_{-\infty}^{+\infty} R\psi^*\, d\tau \qquad (29)$$

Where Re stands for the real part.

By integrating these coupled ordinary differential equations, we will find the evaluation of soliton parameters in the presence of a general perturbation.

The Effect of Raman and Ionization on Soliton Dynamics

In an HC-PCF filled with an ionizable Raman-active gas, pulse propagation is governed by Equation (5):

$$i\frac{\partial\psi}{\partial\xi} + \frac{1}{2}\frac{\partial^2\psi}{\partial\tau^2} + |\psi|^2\psi - \tau_R\psi\frac{\partial|\psi|^2}{\partial\tau} - \eta\psi\int_{-\infty}^{\tau}|\psi|^2 d\tau' = 0$$

This equation can be solved using the multiscale perturbation theory, where the small perturbation is given by:

$$\epsilon R = \tau_R\psi\frac{\partial|\psi|^2}{\partial\tau} + \eta\psi\int_{-\infty}^{\tau}|\psi|^2 d\tau' \quad (30)$$

Substituting (30) for Eqs.(25)-(28) yields the following straightforward coupled ordinary differential equations:

$$A_\xi = 0 \qquad (31)$$

$$\tau_{p\xi} = \delta \qquad (32)$$

$$\delta_\xi = -(\frac{8}{15}\tau_R A^4 - \frac{2}{3}\eta A^2) \qquad (33)$$

$$\phi_\xi = -\frac{1}{2}(\delta^2 - A^2) + \eta A \qquad (34)$$

By introducing a new parameter g from [9,10] and integrating these differential equations, we get the evaluation of soliton parameters as follows,

$$A(\xi) = A(0) = A_0 \qquad (35)$$

$$\tau_p(\xi) = -g\xi^2 \qquad (36)$$

$$\delta(\xi) = -g\xi = \delta_{Raman} + \delta_{ion} \qquad (37)$$

$$\phi(\xi) = -\frac{1}{6}g^2\xi^3 + \frac{1}{2}(A_0 + 2\eta)A_0\xi \quad (38)$$

Where $= g_{red} + g_{blue}$, $g_{red} = \frac{8}{15}\tau_R A_0^4$ and $g_{blue} = -\frac{2}{3}\eta A_0^2$.

Depending on the values of η, τ_R and A_0, g can be positive, negative or zero.

The solution shows clearly that ionization leads to a soliton self-frequency blueshift, opposite to redshift due to Raman self-frequency. In the time domain, ionization effect produces a constant pulse acceleration while Raman effect produces a pulse deceleration.

The results are exactly the same as those obtained by Saleh [9,10].

Conclusion

We began with the perturbed nonlinear Schrödinger equation and applied it to a hollow-core photonic crystal fiber filled with an ionizable gas. Using a multiple scale perturbation theory, the problem was converted to the system of ordinary differential equation. In the present paper we continueed our exact analysis and showed that the resulting formulation yields a new system which satisfies equations (25)-(28). The results obtained in the present paper are just the same obtained via the variational perturbation method [9,10]. Our method shows the appropriateness of multiscale perturbation theory for studying the hollow-core photonic crystal fiber under a general perturbation.

References

[1] Sílvia M.G. Rodrigues, Margarida Facão, Mário F.S. Ferreira, Light propagation in gas-filled kagomé hollow core photonic crystal fibres, Opt. Commun. 412 (2018) 102–107.
[2] S.M.G. Rodrigues, M. Facão, M.F.S. Ferreira, Ultraviolet light generation in gas-filled kagomé photonic crystal fiber, Fiber Integr. Opt. 34 (2015) 76–89.
[3] J. C. Travers, W. Chang, J. Nold, N. Y. Joly, and P. St. J. Russell,Ultrafast nonlinear optics in gas-filled hollow-core photonic crystal fibers [Invited], J. Opt. Soc. Am. B } 28 (2011) A11-A26.
[4] O. H. Heckl, C. R. E. Baer, C. Kr ankel, S. V. Marchese, F. Schapper, M. Holler, T. Sudmeyer, J. S. Robinson, J. W. G.Tisch, F. Couny, P. Light, F. Benabid, and U. Keller,High harmonic generation in a gas-filled hollow-core photonic crystal fiber, Appl.Phys. B 97(2009)369-373.
[5] J. Nold, P. HÖlzer, N. Y. Joly, G. K. L. Wong, A. Nazarkin,A. Podlipensky, M. Scharrer, and P. St. J. Russell,Pressure-controlled phase matching to third harmonic in Ar-filled hollow-core photonic crystal fiber, Opt.Lett. 35 (2010) 2922-2924.
[6] F. Benabid, J. C. Knight, G. Antonopoulos, and P. St. J. Russell, Stimulated Raman scattering in hydrogen-filled hollow-core photonic crystal fiber, Science 298(5592) (2002) 399–402.
[7] D. Ouzounov, C. Hensley, A. Gaeta, N. Venkateraman, M. Gallagher, and K. Koch, Soliton pulse compression in photonic band-gap fibers, Opt. Express 13(16) (2005) 6153-6159.
[8] E. E. Serebryannikov and A. M. Zheltikov, Ionization-induced effects in the soliton dynamics of high-peak-power femtosecond pulses in hollow photonic-crystal fibers, Phys. Rev. A 76 (2007) 013820.
[9] M. F. Saleh, W. Chang, P. HÖlzer, A. Nazarkin, J. C. Travers, N. Y. Joly, P. St. J. Russell, and F. Biancalana,Theory of Photoionization-Induced Blueshift of Ultrashort Solitons in Gas-Filled Hollow-Core Photonic Crystal Fibers, Phys. Rev. Lett. 107 (2011) 203902.
[10] M. F. Saleh, W. Chang, P. Holzer, A. Nazarkin, J. C. Travers, N. Y. Joly, P. St. J. Russell, and Fabio Biancalana, Soliton self-frequency blue-shift in gas-filled hollow-core photonic crystal fibers arXiv preprint arXiv: 1106.5606 (2011)
[11] G. P. Agrawal, Nonlinear Fiber Optics, 4rd ed., Academic Press, 2007.
[12] Ali H. Nayfeh, Perturbation Methods, John Wiley & Sons,1973.
[13] J. P. Keener and D. W. McLaughlin, Solitons under perturbations, Phys. Rev. A 16 (1977) 777.
[14] J. Yang, Multisoliton perturbation theory for the Manakov equations and its applications to nonlinear optics, Phys. Rev. E 59 (1999) 2393–2405.
[15] P. St. J. Russell,Photonic-Crystal Fibers, J. Lightwave Technol. 24 (2006) 4729-4749.