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سالیتون‌های مغناطیسی برای هامیلتونین‌های غیرهایزنبرگی غیرهمسانگرد در برانگیختگی‌های چهارقطبی خطی

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Magnetic Solitons for Non Heisenberg Anisotropic Hamiltonians in Linear Quadrupole Excitations

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چکیده

در این مقاله سیستم با هامیلتونین غیرهایزنبرگی غیرهمسانگرد که دارای تبادل نزدیک‌ترین همسایه است، با استفاده از تقریب میدان میانگین بررسی شده است. ابتدا در حالت کلی معادلاتی که مدل غیرهایزنبرگی غیرهمسانگرد را توصیف می‌کنند با استفاده از حالت‌های همدوس در پارامتر حقیقی محاسبه می‌کنیم و سپس معادلاتی که شاخه‌های دوقطبی و چهارقطبی موج اسپین را برای برانگیختگی‌های خطی کوچک از حالت پایه (خلاء) توصیف می‌کنند، به دست می‌آوریم. در نهایت با استفاده از معادلات خطی شده، جواب سالیتونی برای شاخه چهارقطبی محاسبه می‌کنیم.

واژگان کلیدی

غیرهمسانگرد، غیرهایزنبرگی، برانگیختگی‌های چهارقطبی.

Abstract

We discuss system with non-isotropic non-Heisenberg Hamiltonian with nearest neighbor exchange within a mean field approximation process. We drive equations describing non-Heisenberg non-isotropic model using coherent states in real parameters in general form and then obtain dispersion equations of spin wave of dipole and quadrupole branches for a small linear excitation from the ground state. In final, soliton solution for quadrupole branches for these linear equations obtained.

Keywords

Non - Isotropic, Non - Heisenberg, Quadrupole Excitation.

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Introduction

During the past decade study of nonlinear behavior of magnetic crystals has been attracted large attention, specially it accompanies with the progress in some other fields such as development of theory of nonlinear differential equation, achieving new laboratory results and also potential applications in other branches of science and technology. [1, 2]

Particles with spin $S \geq 1$ are more interesting among the other nano particles [3, 4]. This is because of existing of complexity in their behavior due to their multipole dynamic spin excitations. In such systems, the number of necessary parameters for complete description of macroscopic properties increases up to 4S, that S stands for magnitude of system spin.

Also it worthwhile, the process of achieving classical spin equations and dynamic multipoles is based on coherent states that are obtained in $SU(2S + 1)$ group. [5,6]

We consider unitary anisotropic Hamiltonian as form of:[7]

$$\hat{H} = -J \sum \left(\vec{S}_i \vec{S}_{i+1} + \delta \hat{S}_i^z \hat{S}_i^z \right) \qquad (1)$$

Which, $\hat{\boldsymbol{S}}_{\boldsymbol{i}}^{\boldsymbol{x}}$, $\hat{\boldsymbol{S}}_{\boldsymbol{i}}^{\boldsymbol{y}}$ and $\hat{\boldsymbol{S}}_{\boldsymbol{i}}^{\boldsymbol{z}}$ are spin operators in lattice $\boldsymbol{i}$ and $\boldsymbol{\delta}$ is anisotropic coefficient. This is Hamiltonian of one dimensional ferromagnetic spin chain observed in compositions like $CSNiF_3$.[8]

In this paper the goal is to obtain classical equation for stated Hamiltonian and finding the answer of spin wave for small linear excitations upper than the ground state. Coherent states issued nearest approximation to classical state i.e. pseudo classical, because they minimize uncertainty principles. For this reason, in section 2, coherent states for spin $S =$ 1developed that are the same as coherent states in $SU(3)$ group. To obtain classical Hamiltonian, we need average values of spin operator; so in section 3, these values and classical Hamiltonian equation are derived. In the following, Hamiltonian equation computed in this section is substituted in classical equations of motion resulted from using Feynman path integral on coherent states, and then we acquire spin wave equations and dispersion equations of dipole and quadupole branches for small linear excitation above the ground state, and finally we calculate soliton answers of linearized equations.

Coherent states in $SU(3)$ group

Coherent states are special quantum states that their dynamic is very similar to behavior of their classical system. The kind of coherent state that is used in a problem depends on symmetry of existent operators. With considering existent symmetry in operators of Hamiltonian (1), coherent states in $SU(3)$ group is used for accurate description and considering all multipole excitations. In this group, ground state considered as $(1.0.0)^T$ and its single-site coherent state is written as:[9,10]

$$|\psi\rangle = D^1(\theta.\varphi) e^{-i\gamma \hat{S}^z} e^{2ig\hat{Q}^{xy}} |0\rangle \qquad (2)$$

In above equation, $D^1(\theta.\varphi)$ is Wigner function for spin $S = 1$ and two angles θ and φ determine alignment of classical spin vector. Angle γ determines direction of quadruple excitation around the spin vector. Parameter g specifies change of length of average value of quadruple excitation and also of magnitude of spin vector. Lagrangian can be obtained by use of Feynman path integral for declared coherent states as [11]:

$$L = cos2g(cos\theta\varphi_t + \gamma_t) - H(\theta, \varphi, \gamma) \qquad (3)$$

Where $x_t = \partial / \partial t$ and H is classical energy of system obtained by averaging Hamiltonian (1) on coherent states (2). Two other terms appear when acquiring Lagrangian of spin system. The first is Kinetic term that has Berry phase characteristics issued from quantum interference of Instanton paths and has important role in quantum phenomenons such as spin tunneling and the second is boundary term that depends on boundary values of path. [12] Both of term have no role in classical dynamic of spin excitations and so are not considered here.

3.Classical Hamiltonian and equations in $SU(3)$ group

Average spin values in $SU(3)$ group written as[11]:

$$S^{+} = e^{i\varphi} cos2g sin\theta$$
$$S^{-} = e^{-i\varphi} cos2g sin\theta$$
$$S^{z} = cos2g cos\theta \qquad (4)$$

By averaging Hamiltonian (1) and using (4), the continuous limit of classical Hamiltonian obtained as:

$$H_{cl} = -J\int \frac{dx}{a_0}\Big\{cos^2 2g + \frac{\delta}{2}(cos^2\theta + sin2g cos2\gamma sin^2\theta) - \frac{a_0^2}{2}\big((\theta_x^2 + \varphi_x^2 sin^2\theta)cos^2 2g + 4g_x^2 sin^2 2g\big)\Big\} \qquad (5)$$

To obtain classical equation of motion, the above classical Hamiltonian is substituted in motion equations resulted from Lagrangian equation:

$$\frac{1}{\omega_0}\varphi_t = \delta cos\theta(sec2g - cos2\gamma tan2g) + a_0^2 cos2g(\theta_{xx} csc\theta + \varphi_x^2 cos\theta)$$
$$\frac{1}{\omega_0}\theta_t = \frac{\delta}{2} sin2\theta sin2\gamma tan2g - a_0^2\varphi_{xx} cos2g sin\theta$$
$$\frac{1}{\omega_0} g_t = -\frac{\delta}{2} sin2\gamma sin^2\theta$$
$$\frac{1}{\omega_0}\gamma_t = \{4cos2g - \delta(cos2\gamma(cot4g - cos2\theta csc4g) + cos^2\theta sec2g)\} + \Big\{cos2g\Big(8g_x^2 - 2\theta_x^2 + \frac{1}{2}\varphi_x^2(-3 + cos2\theta) - \theta_{xx}cot\theta\Big) + 4g_{xx} sin2g\Big\} a_0^2 \qquad (6)$$

These equations completely describe nonlinear dynamics of Hamiltonian of problem up to quadrupole excitation. Solutions of these equations are magnetic solitons. These equations result Landau-Lifshitz equation if quadrupole excitations ignored($g = 0$). So these equations are more general in comparison with landau-Lifshitz and have more degree of freedom. It's noteworthy that solution of these equations has different range of solitons.

For small linear excitation from ground stste, classical equations of motion change to:

$$\frac{1}{\omega_0}\varphi_t = \delta(secg_0 + tang_0)\theta + a_0^2 cosg_0\theta_{xx}$$
$$\frac{1}{\omega_0}\theta_t = -a_0^2\varphi_{xx} cosg_0$$
$$\frac{1}{\omega_0} g_t = -\frac{\delta}{2}\gamma$$
$$\frac{1}{\omega_0}\gamma_t = -2\Big(2sing_0 + \frac{\delta}{2cosg_0}\Big) g + 4g_{xx} sing_0 a_0^2 \qquad (7)$$

To obtain dispersion equations, functions $\theta . \varphi . \gamma$ and g are considered as plane waves and their substitution in linearized equations result in dispersion equation for spin wave near the ground state:

$$\omega_1^2 = \omega_0^2 k^2 a_0^2 cosg_0(\delta(secg_0 + tang_0) + k^2 a_0^2 cosg_0)$$
$$\omega_2^2 = \omega_0^2\delta\Big(\frac{\delta}{cosg_0} + 2sing_0(k^2 a_0^2 + 1)\Big) \qquad (8)$$

From the above equation, it is obvious that both dipole and quadruple branches of unitary Hamiltonian are dispersive in presence of linear excitations. If the unitary anisotropy coefficient is zero, ($\delta = 0$), we have only dipole dispersion branch and there is no quadrupole dispersion. In other word, quadruple dispersion branch obtained only when there is square spin term, ($\hat{S}_i^z\hat{S}_i^z$), in Hamiltonian.

To compute soliton answers of equations (7), we define variable η such as $\eta = x - vt$. In this case above equations convert to below nonlinear equations.

$$\Big(\frac{v^2}{\omega_0^2} + \delta a_0^2(1 + sing_0)\Big)\theta_\eta + (a_0^2 cosg_0)^2\theta_{\eta\eta\eta} = 0$$

$$\frac{-2}{\delta\omega_0} g_{tt} = -2\left(2sing_0 + \frac{\delta}{2cosg_0}\right) g + 4a_0^2 g_{xx} sing_0 \quad (9)$$

The first equation is third order differential equation. So change of dipole moment in Hamiltonian (1) is not of the form of soliton. Solution of this equation has the following forms:

$$\theta = Csin\left((x-vt)\left(\frac{(a_0^2 cosg_0)^2}{\left(\frac{v^2}{\omega_0^2}+\delta a_0^2(1+sing_0)\right)}\right)^{1/2}\right) \quad (10)$$

In this answer, if unitary anisotropy coefficient is limited, there is no any change in general form of this, but the magnitude of frequency oscillation changed.

The second equation is nonlinear Klein-Gordon equation and shows change of average value of quadruple excitation that its solution is of the form of Hylomorphic solitons [13]. These solitons are like Q-ball solitons. The reason of this name is because of they cause matter have appropriate form. Also these solitons are of the kind of non topologic ones because their boundary values in ground and infinity are the same from the topological point of view. If rewrite nonlinear Klein-Gordon equation (9) as:

$$g_{tt} = \alpha g_{xx} + \beta g \quad (11)$$

Where

$$\alpha = -\delta\omega_0 a_0^2 sing_0$$
$$\beta = \frac{\delta\omega_0(4sin2g_0+\delta)}{8cosg_0} \quad (12)$$

As we seen in relations (9), (11) and (12), if we remove the unitary anisotropy coefficient δ, the equations are well defined only when $g = 0$. The concept of this sentence is that the quadruple excitation in this equation existed only when there is the anisotropy term in Hamiltonian or in other word quadratic spin term in Hamiltonian.

Numerical solution of (11) is plotted in figure (1). In this computation we consider $\alpha = 10^5$ and $\beta = 10^{10}$.

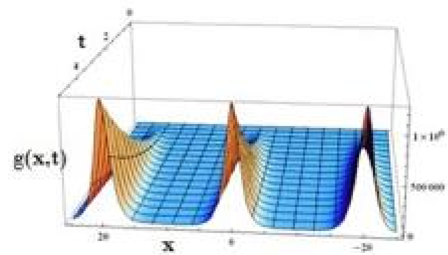

Figure 1. Numerical solution (quadruple excitation) of relation (11) is Helomorphic soliton.

Analytical solution of above nonlinear Klein-Gordon equation is the following form:

$$g(x.t) = C\ \sinh\left[(x - vt)\sqrt{\left|\frac{-\omega_0(4sin2g_0+\delta)}{cosg_0(v^2+\delta\omega_0 a_0^2 sing_0}\right|}\right] \quad (13)$$

Where C is constant.

So, with considering Single-Ion anisotropy in Hamiltonian (1) in SU(3) group, Quadrupole excitations are released as solitary waves that named Hylomorphic soliton and the most property of them is spherical symmetries. This result show the importance of Single-Ion anisotropy in physical system like the Hamiltoniam (1).

Conclusion

In this paper, we study semi-classic theory for spin systems with spin $s = 1$ that contain anisotropic exchange terms. it is shown that for anisotropic ferromagnet, value of average quadruple torque is not constant ($g_t \neq 0$) and its dynamic contains rotational term around classical spin vector ($\gamma_t \neq 0$) and another dynamics that relates to change of length of quadruple torque.

There are no such excitations in regular magnets and their dynamics is achieved by use of average value of Heisenberg spin Hamiltonian. Also it is shown that soliton solutions are of the kind of non topologic Hilomorphic solitons for quadruple excitations.

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